SOPHIA COLLEGE

(AUTONOMOUS)

Affiliated to the University of Mumbai

Syllabi for Semesters I to II

Program : B.Sc

Course: Mathematics

(Choice Based Credit System with effect from the year 2021-22)

FYBSc Mathematics SEMESTER I

CALCULUS I				
Course Code	Unit	Topics	Credits	L/Week
SBSMTT101	I	Real Number System	2	3
	II	Limits of Real Valued Functions		
	III	Continuous Functions		
Algebra and Discrete Mathematics- I				
SBSMTT102	I	Integers and divisibility	2	3
	II	Binary Operations, Equivalence Relations and Congruences		
	III	Polynomials		
SBSMTTP1		PRACTICALS	2	2

FYBSc Mathematics SEMESTER II

CALCULUS II				
Course Code	Unit	Topics	Credits	L/Week
SBSMTT201	I	Differentiation of real valued function of one variable	2	3
	II	Applications of differentiation		
	III	Mean Value Theorem and their applications.		
Algebra and Discrete Mathematics -II				
SBSMTT202	I	Recurrence relations and Counting Problems	2	3
	II	Counting Principles		
	III	Permutations, Principle of Inclusion-Exclusion		
SBSMTTP2		PRACTICALS	2	2

The Mathematics course is offered at the First Year Level of the Under-graduate Science programme. The student opts for two papers in each of the two semesters.

Learning Objectives:

1. To develop in the learner, critical thinking and logical reasoning.
2. To enable the learner to understand the abstract mathematical concepts
3. To develop in the learner, numerical and analytical skills.

FYBSc Maths Paper I Semester I
CALCULUS I

Course Objectives:

1. To develop in the learner an understanding of the structure of the real number system.
2. To develop in the learner an understanding of the characteristics of various mathematical functions and plot their graphs.
3. To enable in the learner the knowledge of the concepts of limits of a function and the nature of continuous functions.

Unit I: Real Number System (15 Lectures)

i. Real number system R and order properties of R, Absolute values and its properties.
ii. AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, Hausdorff property.
iii. Bounded sets, l.u.b and g.l.b, l.u.b. axiom and its consequences, Archimedean property and its applications, density of rationals and irrationals.

Unit II: Limits of a real valued function (15 Lectures)

i. Definitions – Function, domain and range of a function, direct image and inverse image of a function *f*, bounded functions, injective function, surjective function, bijective function, composite of two functions (when defined), inverse of a bijective function.
ii. Graphs of some standard functions such as $|x|$; e^x; log x; ax^2+bx+c; 1/x, x^n $(n \leq 3)$; sin x; cos x; tan x; x sin(1/x); x^2 sin(1/x), step functions over suitable intervals of R:
iii. Definition and examples of limit of a function, left-hand-limit, right-hand-limit, uniqueness of limit if it exists, algebra of limits, limit of composite function, sandwich theorem, non-existence of limits

Unit III: Continuous functions: (15 Lectures)

i. Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain.
ii. Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.
iii. Intermediate value theorem and its applications, Bolzano-Weierstrass theorem; Continuity on closed and bounded intervals.

Course Outcomes

1. The learner should be able to understand the structure of the real number system.
2. The learner is able to understand the characteristics of the functions and plot their graphs.
3. The learner can calculate the limits of a function and find the nature of continuous functions.

Main Reference:

1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte. Ltd.
2. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
3. Ajit Kumar-S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

Additional Reference Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R.G. Bartle- D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
4. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
5. Ghorpade, Sudhir R.- Limaye, Balmohan V., A Course in Calculus and Real Analysis, Springer International Ltd, 2000.
6. G.B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison Wesley, 1998.

FYB.Sc Maths Paper II Semester I
Algebra and Discrete Mathematics I

Course Objectives:

1. To develop in the learner an understanding of the discrete number system.
2. To develop in the learner an understanding of the characteristics of various mathematical relations and their applications.
3. To enable in the learner the knowledge of polynomials and their properties.
4. To enable the learner to understand the applications of complex numbers in polynomials.

Prerequisites:

Set Theory: Set, subset, set union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, relations, Permutations and Combinations, ${}_nP_r$ and ${}_nC_r$.

Complex Numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number

Unit I: Integers and divisibility (15 Lectures)

i. Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal's Triangle.
ii. Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m) of two integers, basic properties of gcd such as existence and uniqueness of g.c.d. of integers *a* and *b*, g.c.d can be expressed as *ma+nb, m,n* are integers. Euclid's lemma, Euclidean algorithm.
iii. Results on prime numbers and fundamental theorem of arithmetic.

Unit II: Binary operations, Equivalence Relations and Congruence (15 Lectures)

i. Definition of relation and function, Binary operations as a function, properties and examples.
ii. Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.
iii. Congruence - definition, elementary properties and applications. Euler's φ function, Statements of Euler's theorem, Fermat's theorem and Wilson theorem and their applications
iv. Congruence as an equivalence relation on Z (set of integers), Residue classes and its properties. partition of Z, Addition modulo *n*, Multiplication modulo *n*, examples.

Unit III: Polynomials (15 lectures)

i. Definition of polynomial, Polynomials over F where F = Q or R, Algebra of polynomials, basic properties, division algorithm in F[X] and g.c.d of two polynomials and its basic properties, Euclidean algorithm, applications,
ii. Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, remainder theorem, Factor theorem, applications, A polynomial of degree *n* has at most *n* roots, Complex and non-real roots of a polynomials in R[*X*] occur in conjugate pairs.
iii. Necessary conditions for a rational number p/q to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is not a rational number where p is a prime number.

iv. Complex numbers - DeMoivres Theorem, roots of unity, primitive roots of unity, solutions of the equation $w^n = z$. Fundamental theorem of algebra, roots of polynomials over R.

Course Outcomes:

1. The learner understands the structure of the discrete number system and the various properties on divison in integers.
2. The learner will be able to identify the various types of relations on discrete systems.
3. The learner will be able to work on various binary operations of polynomials and find the roots of the polynomials.
4. The learner will be able to apply the concepts of complex numbers in polynomials.

Main Reference:

1. Elementary Number Theory, David M. Burton, Second Edition, UBS, New Delhi.
2. Discrete Mathematics, Norman L. Biggs, Revised Ed, Clarendon Press, Oxford 1989.
3. A Foundation Course in Mathematics- Ajit Kumar, S. Kumaresan, Bhaba Sarma, Narosa

Additional Reference Books:

1. K.D. Joshi, Foundations in Discrete Mathematics, New Age Publishers, New Delhi, 1989.
2. Kenneth H. Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition.
3. Norman Biggs: Discrete Mathematics, Oxford.

FYB.Sc Maths Paper I Semester II
CALCULUS II

Course Objectives:

1. To develop in the learner, an understanding of the concepts of derivative of a function.
2. To impart knowledge of the methods of finding the higher order derivative of the given function.
3. To enable the learner understands the applications of the derivative of a function.
4. To develop an understanding of the concepts and application of Mean Value theorems.

Unit I : Differentiation of real valued function of one variable: **(15 Lectures)**

i. Definition of differentiation at a point of an open interval, examples of differentiable and non-differentiable functions, relation between continuity and differentiability.
ii. Algebra of differentiable functions. Chain rule, Derivative of inverse functions, Implicit differentiation
iii. Higher order derivatives, Leibnitz rule for higher order derivatives.

Unit II: Applications of differentiation **(15 Lectures)**

i. Increasing and decreasing functions, definition of local maximum and local minimum, stationary points, first and second derivative test, examples,
ii. Graph of functions using first and second derivatives, concave functions, points of inflection.
iii. Geometric Interpretation of Derivatives- applications such as rate of change in area and volume

Unit III: Mean Value Theorems and their Applications **(15 Lectures)**

i. Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples.
ii. Taylor's theorem and its applications.
iii. L-hospital's rule without proof, examples of indeterminate forms.

Course Outcomes:

1. The learner can find the derivative of a function on the set of real numbers.
2. The learner will be able to find the higher order derivatives of the functions
3. The learner will be able to apply the various concepts of differentiation on the functions to find the nature of the function.
4. The learner will be able to apply the concepts of Mean Value theorems and find the approximate value of the function at a certain point.

Main Reference:

1. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
2. G.B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley.
3. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

Additional Reference Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R.G. Bartle- D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
4. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
5. Ghorpade, Sudhir R.- Limaye, Balmohan V., A Course in Calculus and Real Analysis, Springer International Ltd, 2000.

FYB.Sc Maths Paper II Semester II
Algebra and Discrete Mathematics II

Course Objectives:

1. To understands the concepts of recurrence relations and apply the results in the counting problems.
2. To enable the learner to apply the various types of counting principles on discrete systems.
3. To develop an understand the concepts and applications of permutation maps on discrete sets .
4. To apply the various concepts of counting principles along with the principle of Inclusion-Exclusion and find solutions to the problems based on them.

Unit I Recurrence Relations and Counting problems (15 Lectures)

i. Recurrence Relations, definition of homogeneous, non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relations of Tower of Hanoi, Fibonacci sequence, etc. in counting problems.
ii. Solving homogeneous as well as non-homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.
iii. Counting problems using tree diagrams.

Unit II: Counting Principles (15 lectures)

i. Finite and infinite sets, countable and uncountable sets examples such as N, Z, N × N, Q, (0, 1), R.
ii. Addition and multiplication principles, distributions of distinct and non-distinct objects, Multinomial coefficients, combinatorial interpretations, Multinomial theorem, applications.
iii. Pigeonhole Principle and its applications.

Unit III: Permutations, Principle of Inclusion-Exclusion (15 lectures)

i. Permutations of {1,2,…..n}. Cycles and transpositions. Decomposition of a permutation as a product of disjoint cycles and as product of transpositions. Inversions in a permutation. Sign of a permutation. Even and odd permutations.
ii. Principle of inclusion and exclusion, its applications, derangements, explicit formula for d_n.
iii. Stirling number S(n, k) and its elementary properties,

Course Objectives:

1. The learner will understand the results of recurrence relations and apply the results in the counting problems.
2. The learner will be able to apply the various types of counting principles on discrete systems.
3. The learner will understand the concepts and results of permutation maps and apply them to different mathematical problems on discrete sets .
4. The learner will connect various concepts of counting principles along with the principle of Inclusion-Exclusion and find solutions to the problems based on them.

Main Reference:

1. Elementary Number Theory, David M. Burton, Second Edition, UBS, New Delhi.

2. Kenneth H. Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition.
3. Discrete Mathematics, Norman L. Biggs, Revised Edition, Clarendon Press, Oxford 1989.

Additional Reference Books:

1. K.D. Joshi, Foundations in Discrete Mathematics, New Age Publishers, New Delhi, 1989.

Suggested List of Practical (CALCULUS I)

1. Application of Archimedean property, intervals, neighbour-hood.
2. Consequences of l.u.b. axiom, infimum and supremum of sets.
3. Functions.
4. Limits, finding the Left- and Right-hand limit of the function
5. Continuous and discontinuous functions.
6. Applications of Intermediate Value theorem

Suggested List of Practical (Algebra and Discrete Mathematics I)

1. Principle of finite induction
2. Binomial theorem
3. G.C.D of an integer
4. Binary operations. equivalence relation and equivalence classes
5. Congruence modulo relation on integers
6. Division of polynomials; gcd; roots of a polynomial – real and compex.

Suggested List of Practical (CALCULUS II)

1. Differentiability of a function,
2. Higher order derivatives, Leibnitz theorem.
3. Maxima, Minima and points of inflections
4. L'Hospitals Rule
5. Mean value theorems and its applications.
6. Applications of Taylor's theorem and Taylor's polynomials.

Suggested List of Practical (Algebra and Discrete Mathematics II)

1. Recurrence relations.
2. Counting problems using trees.
3. Counting sets, Addition and Multiplication principles
4. Multinomial Theorem and Pigeonhole principle
5. Permutations and derangements
6. Principle of Inclusion exclusion and Stirling's number

Pattern Paper:

Internal Assessment :

The IA consists of two tests/projects of 25 marks. The total of the marks for Internal Assessment is 50.

Semester End Examination (Theory):

At the end of the semester, Theory examination for 50 marks based on the three units shall be held for each course.

Pattern of **Theory question** paper at the end of the semester for **each course:**

1. There shall be four questions, first three questions shall be of 20 marks on each unit and fourth question will be of 15 marks based on Unit I, II and Unit III.
2. All questions shall be compulsory with internal choice within the questions.

Questions	Sub-questions	Maximum marks
Q1	Part A: two theory sub-questions each one is of 8 marks and attempt any one. Part B: Four sub-questions, each one is of 4 marks and attempt any three.	20 each
Q2		
Q3		
Q4	There shall be 6 sub-questions each one is of 5 marks and attempt any 3.	15
Total marks		75