
# SOPHIA COLLEGE

# (AUTONOMOUS)

Affiliated to the University of Mumbai

Syllabi for Semesters I to II

Program : B.Sc

Course: Mathematics

(Choice Based Credit System with effect from the year 2018-19)

**SEMESTER I**

| CALCULUS I | | | | |
|---|---|---|---|---|
| Course Code | Unit | Topics | Credits | L/Week |
| SBSMTT101 | I | Real Number System | 2 | 3 |
| | II | Limits & Continuity | | |
| | III | Continuous functions | | |
| **Algebra and Discrete Mathematics- I** | | | | |
| SBSMTT102 | I | Integers and divisibility | 2 | 3 |
| | II | Equivalence Relations and Congruences | | |
| | III | Functions and Binary operations | | |
| SBSMTTP1 | | PRACTICALS | 2 | 2 |

**SEMESTER II**

| CALCULUS II | | | | |
|---|---|---|---|---|
| Course Code | Unit | Topics | Credits | L/Week |
| SBSMTT201 | I | Differentiation of real valued function of one variable | 3 | 3 |
| | II | Applications of differentiation | | |
| | III | Analytic Geometry in Euclidian Spaces | | |
| **Algebra and Discrete Mathematics -II** | | | | |
| SBSMTT202 | I | Polynomials | 3 | 3 |
| | II | Counting Principles | | |
| | III | Complex Numbers | | |
| SBSMTTP2 | | PRACTICALS | 2 | 2 |

List of Course Titles and Course Codes

| Class | Sem | Paper No. | Course Title | Course Credits | Course Codes |
|---|---|---|---|---|---|
| FYBSc | I | I | Calculus I | 2 | SBSMTT101 |
| FYBSc | I | II | Algebra and Discrete Mathematics I | 2 | SBSMTT102 |
| FYBSc | I | | Practicals | 2 | SBSMTTP1 |
| FYBSc | II | I | Calculus II | 2 | SBSMTT201 |
| FYBSc | II | II | Algebra and Discrete Mathematics II | 2 | SBSMTT202 |
| FYBSc | II | | Practicals | 2 | SBSMTTP2 |

**Department**: Maths & Statistics (Mathematics) **Year**: 2018-19

Head of Department _____

**FYBSc Maths Paper I Semester I**
**CALCULUS I**

**Unit I: Real Number System (15 Lectures)**

i. Real number system R and order properties of R, Absolute values and its properties.
ii. AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, Hausdorff property.
iii. Bounded sets, l.u.b and g.l.b, l.u.b. axiom and its consequences, Archimedean property and its applications, density of rationals and irrationals.

**Unit II: Limits of a real valued function (15 Lectures)**

i. Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function.
ii. Graphs of some standard functions such as $|x|$; $e^x$; $\log x$; $ax^2+bx+c$; $1/x$, $x^n$ $(n \geq 3)$; $\sin x$; $\cos x$; $\tan x$; $x \sin(1/x)$; $x^2 \sin(1/x)$, step functions over suitable intervals of R:
iii. Definition and examples of limit of a function ,left-hand-limit, right-hand-limit, uniqueness of limit if it exists, algebra of limits , limit of composite function, sandwich theorem, non-existence of limits

**Unit III: Continuous functions: (15 Lectures)**

i. Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain.
ii. Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.
iii. Intermediate value theorem and its applications, Bolzano-Weierstrass theorem; Continuity on closed and bounded intervals.

**Main Reference:**


1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte. Ltd.
2. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
3. Ajit Kumar-S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.


**Additional Reference Books**:


1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R.G. Bartle- D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
4. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
5. Ghorpade, Sudhir R.- Limaye, Balmohan V., A Course in Calculus and Real Analysis, Springer International Ltd, 2000.
6. G.B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison Wesley, 1998.

**FYB.Sc Maths Paper II Semester I**
**Algebra and Discrete Mathematics I**

**Prerequisites:**
**Set Theory**: Set, subset, set union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, relations, Permutations and Combinations, ${}_nP_r$ and ${}_nC_r$.
**Complex Numbers**: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number

**Unit I: Functions and Binary operations (Unit I) (15 Lectures)**

i. Definition of function, domain, co-domain and range of a function, composite function, examples, direct image and inverse image of a function *f*, injective, surjective and bijective functions, invertible functions.
ii. Binary operations as a function, properties and examples.

**Unit II: Integers and divisibility (15 Lectures)**

i. Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal's Triangle.
ii. Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m) of two integers, basic properties of gcd such as existence and uniqueness of g.c.d. of integers *a* and *b*, g.c.d can be expressed as *ma+nb, m,n* are integers. Euclid's lemma, Euclidean algorithm.
iii. Results on prime numbers and fundamental theorem of arithmetic.

**Unit III: Equivalence Relations and Congruences (15 Lectures)**

i. Equivalence relations, Equivalence classes, properties such as two equivalence classes are either identical or disjoint, definition of partition, every partition gives rise of an equivalence relation and vice versa.
ii. Congruence - definition , elementary properties and applications. Euler's φ function, Statements of Euler's theorem, Fermat's theorem and Wilson theorem and their applications
iii. Congruence as an equivalence relation on Z (set of integers), Residue classes and its properties.

**Main Reference:**


1. Elementary Number Theory, David M. Burton, Second Edition, UBS, New Delhi.
2. Discrete Mathematics, Norman L. Biggs, Revised Edition, Clarendon Press, Oxford 1989.
3. A Foundation Course in Mathematics- Ajit Kumar, S. Kumaresan, Bhaba Sarma, Narosa


**Additional Reference Books**:


1. K.D. Joshi, Foundations in Discrete Mathematics, New Age Publishers, New Delhi, 1989.
2. Kenneth H. Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition.
3. Norman Biggs: Discrete Mathematics, Oxford.

**FYB.Sc Maths Paper I Semester II**
**CALCULUS II**

**Unit I : Applications of differentiation** **(15 Lectures)**

i. Increasing and decreasing functions, definition of local maximum and local minimum , stationary points, first and second derivative test, examples, Graph of functions using first and second derivatives, concave functions, points of inflection.
ii. Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples.
iii. L-hospital rule without proof, examples of indeterminate forms.
iv. Taylor's theorem and its applications.

**Unit II: Analytic Geometry in Euclidean Spaces** **(15 Lectures)**

i. Review of vectors in $R^2$ and $R^3$, component form of vectors, basic notions such as addition and scalar multiplication of vectors, dot product of vectors, orthogonal vectors, length (norm ) of a vector , unit vector, distance between two vectors, cross product of vectors in $R^3$, scalar triple product (box product), vector projections
ii. Lines and planes in space, equation of sphere, cylinders and quadric surfaces (identification and graph)
iii. Coordinate systems in $R^2$ and $R^3$

**Unit III: Differentiation of real valued function of one variable**: **(15 Lectures)**

i. Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable functions, relation between continuity and differentiability.
ii. Algebra of differentiable functions. Chain rule, Derivative of inverse functions, Implicit differentiation
iii. Higher order derivatives, Leibnitz rule for higher order derivatives.

**Main Reference:**


1. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
2. G.B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley.
3. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.


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1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R.G. Bartle- D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
4. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
5. Ghorpade, Sudhir R.- Limaye, Balmohan V., A Course in Calculus and Real Analysis, Springer International Ltd, 2000.

# FYB.Sc Maths Paper II Semester II
# Algebra and Discrete Mathematics II

**Unit I: Polynomials** **(15 lectures)**

i. Definition of polynomial, Polynomials over F where F = Q or R, Algebra of polynomials, basic properties, division algorithm in F[X] and g.c.d of two polynomials and its basic properties, Euclidean algorithm, applications,
ii. Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, remainder theorem, Factor theorem, applications, Necessary conditions for a rational number p/q to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is not a rational number where p is a prime number.

**Unit II: Counting Principles** **(15 lectures)**

i. Addition and multiplication principles, distributions of distinct and non-distinct objects, Multinomial coefficients, combinatorial interpretations, Multinomial theorem, applications
ii. Pigeonhole Principle and its applications.
iii. Stirling number S(n, k) and its elementary properties.
iv. Permutations of {1,2,…..n}. Cycles and transpositions. Decomposition of a permutation as a product of disjoint cycles and as product of transpositions. Inversions in a permutation. Sign of a permutation. Even and odd permutations, derrangements.

**Unit III: Complex Numbers** **(15 lectures)**

i. Addition and multiplication of complex numbers, modulus and amplitude of a complex number, real and imaginary parts and the conjugate of a complex number.
ii. Geometric representation of complex numbers, modulus, amplitude and conjugate of a complex number.
iii. DeMoivres Theorem, roots of unity, primitive roots of unity, solutions of the equation $w^n = z$.
iv. Fundamental theorem of algebra, roots of polynomials over R.

**Main Reference:**


1. Elementary Number Theory, David M. Burton, Second Edition, UBS, New Delhi.
2. Kenneth H. Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition.
3. Discrete Mathematics, Norman L. Biggs, Revised Edition, Clarendon Press, Oxford 1989.


**Additional Reference Books**:


1. K.D. Joshi, Foundations in Discrete Mathematics, New Age Publishers, New Delhi, 1989.

**Suggested List of Practical (CALCULUS I)**

1. Application of Archimedean property, intervals, neighbour-hood.
2. Consequences of l.u.b. axiom, infimum and supremum of sets.
3. Limit of a function and Sandwich theorem.
4. Left and Right hand limit
5. Continuous and discontinuous functions.
6. Applications of Intermediate Value theorem

**Suggested List of Practical (Algebra and Discrete Mathematics I)**

1. Principle of finite induction
2. Binomial theorem
3. G.C.D of an integer
4. Equivalence relation and equivalence classes
5. Congruence modulo relation on integers
6. Functions and binary operations

**Suggested List of Practical (CALCULUS II)**

1. Differentiability of a function,
2. Higher order derivatives, Leibnitz theorem.
3. Maxima , Minima and points of inflections
4. Mean value theorems and its applications.
5. Applications of Taylor's theorem and Taylors polynomials.
6. Lines and planes in space

**Suggested List of Practical (Algebra and Discrete Mathematics II)**

1. Division of polynomials and gcd
2. Root of a polynomial
3. Pigeonhole principle
4. Permutations and derangements
5. Inclusion exclusion
6. Complex numbers

## Pattern Paper:

### Internal Assessment :

The IA consists of test/project of 20 marks and class participation of 5 marks.

**Semester End Examination (Theory):**

At the end of the semester, Theory examination of 2.5 hours duration and 75 marks based on the three units shall be held for each course.

Pattern of **Theory question** paper at the end of the semester for **each course:**

1. There shall be four questions, first three questions shall be of 20 marks on each unit and fourth question will be of 15 marks based on Unit I, II and Unit III.
2. All questions shall be compulsory with internal choice within the questions.

| Questions | Sub-questions | Maximum marks |
|---|---|---|
| Q1 | Part A: two theory sub-questions each one is of 8 marks and attempt any one.<br>Part B: Four sub-questions, each one is of 4 marks and attempt any three. | 20 each |
| Q2 | | |
| Q3 | | |
| Q4 | There shall be 6 sub-questions each one is of 5 marks and attempt any 3. | 15 |
| Total marks | | 75 |